

ORDERED BINARY SHIFTS WITH A HOLE

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ABSTRACT. In several dynamical systems on a real interval (doubling map with a hole, Lorenz maps, unique beta-expansions, ...), the orbits are coded by binary sequences avoiding an interval. Usually, the interval of sequences is taken with respect to the lexicographic order, and we know from Labarca and Moreira (2006) when a binary shift with a lexicographic hole has positive entropy. With Komornik and Zou (2024), I formulated this result in terms of S-adic sequences involving the Thue-Morse and Sturmian substitutions. The aim of this talk is to extend the characterisation of positive entropy to other piecewise monotonic orders on sequences, namely the alternating lexicographic order, the tent map order and its inverse (which correspond to the negative doubling map, the tent map and the V map). We obtain again a characterisation in terms of compositions of a finite set of substitutions, but now we have a graph-directed construction instead of arbitrary compositions of substitutions. Moreover, renormalisation for one piecewise monotonic order can lead to another one, so that the characterisations for the 4 orders are interconnected.